pulse; $\Delta T_{cool\ per}$, the permissible residual overheating of the thermistor at the end of the interval between pulses; η, thermistor coefficient of thermal loading; τ_e, electrical time constant of thermistor; ΔR_T, increment of thermistor resistance due to action of voltage pulse.

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DETERMINATION OF THE MEAN ENERGY DENSITY OF A LIGHT BEAM IN AN IRREGULAR THERMODYNAMIC LIGHT GUIDE

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The mean energy density of a beam of light in an irregular thermodynamic light guide with random lens shifts is calculated using the approach described in [3].

§1. The energy structure of a beam of light propagating in an optical communication line consisting of a series of discrete phase correctors is usually studied by quasioptical methods [1]. For regular lines this approach enables one to obtain fairly complete information on the mode structure, the losses in the line, etc., but serious difficulties are encountered when one attempts to apply a similar analysis to lines which have a different kind of statistical irregularity (displacement of correctors, rotation of the beam, differences between the corrector parameters, etc.).

Geometrical optics, which is simpler than other approaches, enables one to obtain reasonable information on the energy distribution in the beam of light. One of the versions of this approach is the ray method (see [2] and the references given there). Another approach by which the energy distribution can be analyzed using geometrical optics has been described in [3]. A differential equation for the light energy density was obtained there which enables one to find the energy density at any point in the region for an assigned initial distribution. The evolution of the energy distribution of the beam in this approach is traced in phase space of the beam, and the energy density is therefore a function of the vector which defines the coordinates of the point in space and the vector which defines the direction at the same point. The main result obtained in [3] is that the energy density $U(x, p_i, q_i)$ at a point M with coordinates (x, q_i) in the direction (p_i) is determined by the initial energy

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density $\Phi[q_i^0(x, q_k, p_k), p_i^0(x, q_k, p_k)]$, where q_i^0 and p_i^0 are expressed in terms of x, q_k, and p_k using the dynamic equations ($i, k=1, 2, \ldots, n-1$, where n is the dimension of the space).

In the present paper, using the approach developed in [3], we analyze the energy of a beam of light propagating through a system of aberrationless thermal gas lenses subjected to stochastic transverse displacements.

§ 2. We will assume that the optical system possesses axial symmetry and we will therefore only consider the plane case. We will take as a model of the light guide a system of N similar thermal gas lenses of length l, each of which can be independently displaced by a random amount from the ideal optic axis x and are separated by optically uniform sections of length D. We will assume that the random transverse displacements of the k-th lens λ_k obey a definite law (for example, a normal law) with a distribution function $\rho(\lambda_k)$. Generally speaking, it is unnecessary to assume that there is no correlation between the lens displacements: the whole system can be characterized by the distribution function $\rho(\lambda_1, \lambda_2, \ldots, \lambda_N)$, but we will assume in practice that $\rho(\lambda_1, \ldots, \lambda_N) = \prod_{k=1}^{N} \rho(\lambda_k)$. We will also assume that the diameter of a lens is much greater than the diameter of the entering beam and that the variance of the random displacements σ is small; we will therefore neglect the finite size of the lens apertures.

It was shown in [3] that the energy density of a beam of light propagating in a medium with a refractive index of the form $n = n_0(1 - \frac{\alpha}{2} y^2)$* is given by the expression

$$U(x, y, \dot{y}) = \Phi\left[y\cos\sqrt{\alpha}x - \frac{\dot{y}}{\sqrt{\alpha}}\sin\sqrt{\alpha}x, \right. \\ \left. - y\sqrt{\alpha}\sin\sqrt{\alpha}x + \dot{y}\cos\sqrt{\alpha}x \right]. \tag{1}$$

Here x and y are the longitudinal and transverse coordinates, respectively, and $\dot{y} = dy/dx$. It follows from Eq. (1) that the energy density at the exit of a lens of length l is

$$U(x, y, \dot{y}) = \Phi\left[y\cos\sqrt{\alpha}l - \frac{\dot{y}}{\sqrt{\alpha}}\sin\sqrt{\alpha}l, \right. \\ \left. y\sqrt{\alpha}\sin\sqrt{\alpha}l + \dot{y}\cos\sqrt{\alpha}l \right]. \tag{2}$$

We will now introduce the following notation:

$$\varphi = \sqrt{\alpha}l,\ L = \sqrt{\alpha}D,\ \eta = \dot{y}/\sqrt{\alpha},\ Y = \begin{pmatrix} y \\ \eta \end{pmatrix},\ \Lambda = \begin{pmatrix} \lambda \\ 0 \end{pmatrix}, \\ V = \sigma_0\cos\varphi - i\sigma_2\sin\varphi,\ P = \sigma_0 - \frac{L}{2}(\sigma_1 + i\sigma_2). \tag{3}$$

Here σ_0 is a unit 2×2 matrix and σ_i is the Pauli matrix, which satisfy the well-known relation

$$\sigma_i\sigma_k = \sigma_0\delta_{ik} + i\varepsilon_{ikl}\sigma_l, \tag{4}$$

where δ_{ik} is the Kronecker delta and ε_{ikl} is the Levi-Civita tensor. In the notation of Eqs. (3) the exit coordinates of the beam at the k-th lens Y_k are related to the entrance coordinates of the beam at the (k+1)-st lens $Y_{k+1,0}$ by the matrix P:

$$Y_k = PY_{k+1,0}, \tag{5}$$

while the entrance coordinates of the beam at the k-th lens $Y_{k,0}$ are related to its exit coordinates by the relation

$$Y_{k,0} = VY_k + (1 - V)\Lambda_k. \tag{6}$$

The sequential use of recurrent relations of the form (5) and (6) enables us to relate the entrance coordinates of the beam at the first lens to its exit coordinates at the N-th lens:

$$Y_{1,0} = (VP)^{N-1}VY_N + \sum_{k=0}^{N-1}(VP)^k(1 - V)\Lambda_{k+1}. \tag{7}$$

* This refractive index for $\alpha \ll 1$ corresponds to an aberrationless thermal gas lens [4].

The expression obtained enables us to calculate the mean-square deviation of the beam in the system [5].

Hence, using the approach described in [3] we obtain the dynamic distribution of the energy density at the exit of the N-th lens:

$$U(y_N, \eta_N, \lambda_1, \ldots \lambda_N) = \Phi[(Y_{1,0})_1, (Y_{1,0})_2] \equiv \Phi(Y_{1,0}), \tag{8}$$

where we must take as $(Y_{1,0})_i$ the corresponding components of the representation (7).

It can be shown that sequential matching of the distribution Φ at each boundary of separation and direct calculation of the trajectory [Eq. (7)] lead to the same result for the dynamic energy distribution (8). This is not so in the case of a finite lens aperture: suppose $2a$ is the diameter of the lens; then introducing the vector $A = \begin{pmatrix} a \\ 0 \end{pmatrix}$, we obtain for the dynamic energy density distribution at the exit of the system instead of Eq. (8) the expression

$$\begin{aligned} U(y_N, \eta_N, \lambda_1, \ldots, \lambda_N) = \Phi\Big[(VP)^{N-1} VY_N + \sum_{k=0}^{N-1} (VP)^k (1-V)\Lambda_{k+1} \Big] \times \\ \times \prod_{m=0}^{N-1} \theta\Big[(VP)^{N-m-1} VY_N + \sum_{k=m}^{N-1} (VP)^{k-m} (1-V)\Lambda_{k+1} + \\ + A - \Lambda_{m+1} \Big] \Big[A - \Lambda_{m+1} - (VP)^{N-m-1} VY_N - \sum_{k=m}^{N-1} (VP)^{k-m} (1-V)\Lambda_{k+1} \Big]. \end{aligned} \tag{9}$$

The Heaviside function θ is understood here in the sense

$$\theta(Y) = \theta\left[\begin{pmatrix} y \\ \eta \end{pmatrix}\right] \equiv \theta(y) = \begin{cases} 1, & y \geqslant 0, \\ 0, & y < 0. \end{cases}$$

In order to obtain the mean energy density distribution in the beam passing through the above lens system we must average the dynamic distribution (8) or (9) with the distribution function $\rho(\lambda_1, \lambda_2, \ldots, \lambda_N)$ of the random lens displacements:

$$\begin{gathered} \bar{U}(y_N, \eta_N) = \frac{1}{\|\rho\|} \int d\lambda_1 \ldots d\lambda_N \rho(\lambda_1, \ldots, \lambda_N) U(\lambda_k, y_N, \eta_N), \\ \|\rho\| = \int d\lambda_1 \ldots d\lambda_N \rho(\lambda_1, \ldots, \lambda_N). \end{gathered} \tag{10}$$

§ 3. Let us consider once again Eq. (8) and put

$$y_N = y, \ \eta_N = \eta, \ (VP)^{N-1} V = \|a_{il}\|, \ (VP)^{k-1}(1-V) = \|b_{il}^k\|;$$

then λ_k and b_{il}^k can be regarded as components of N-dimensional vectors $\lambda = (\lambda_1, \lambda_2, \ldots, \lambda_N)$ and $b_{il} = (b_{il}^1, b_{il}^2, \ldots, b_{il}^N)$. In the expression for $U(y, \eta, \lambda)$ we have the terms $\sum_{k=1}^{N} b_{11}^k \lambda_k$ and $\sum_{k=1}^{N} b_{21}^k \lambda_k$, which can be more conveniently represented by (g_1, λ) and (g_2, λ), respectively; then

$$U(y, \eta, \lambda) = \Phi[a_{11}y + a_{12}\eta + (g_1, \lambda), \ a_{21}y + a_{22}\eta + (g_2, \lambda)] \tag{11}$$

and the average energy density is

$$\bar{U}(y, \eta) = \frac{1}{\|\rho\|} \int d\lambda \rho(\lambda) \Phi[a_{11}y + a_{12}\eta + (g_1, \lambda), \ a_{21}y + a_{22}\eta + (g_2, \lambda)]. \tag{12}$$

The integral in Eq. (12) is N-dimensional and is inconvenient for practical calculations, but in a number of cases it can be represented in compact form because $U(y, \eta, \lambda)$ depends on the scalar product of the vectors g_i and λ.

We will obtain this compact representation for $\bar{U}$ assuming that $\Phi(y_{1,0}, \eta_{1,0})$ is an analytical function of its variables and can be expanded in a uniformly converging power series:

$$\Phi \equiv F[y, \eta, (g_1, \lambda), (g_2, \lambda)] = \sum_{m, n=0}^{\infty} c_{mn}(y, \eta)(g, \lambda)^m (g_2, \lambda)^n, \tag{13}$$

and the distribution function of the random displacements ρ is a Gaussian distribution with variance σ:

$$\rho(\lambda) = (2\pi\sigma)^{-N/2} \exp\left[-\frac{(\lambda, \lambda)}{2\sigma}\right]. \tag{14}$$

We will introduce the generating functional K,

$$K(z_1, z_2, g_1, g_2) = \frac{1}{\|\rho\|} \int d\lambda \rho(\lambda) \exp[iz_1(g_1, \lambda) + iz_2(g_2, \lambda)], \tag{15}$$

and we will apply to it the formal operator series

$$F\left(y, \eta, \frac{1}{i}\frac{\partial}{\partial z_1}, \frac{1}{i}\frac{\partial}{\partial z_2}\right) = \sum_{m, n=0}^{\infty} c_{mn}(y, \eta) \left(\frac{1}{i}\frac{\partial}{\partial z_1}\right)^m \left(\frac{1}{i}\frac{\partial}{\partial z_2}\right)^n. \tag{16}$$

Using the uniform convergence of the series, we can write

$$\bar{U}(y, \eta) = \left[F\left(y, \eta, \frac{1}{i}\frac{\partial}{\partial z_1}, \frac{1}{i}\frac{\partial}{\partial z_2}\right) K(z_1, z_2, g_1, g_2)\right]_{z_1=z_2=0}. \tag{17}$$

It is easy to calculate the functional K for the distribution ρ of the form (14):

$$K(z_1, z_2, g_1, g_2) = \exp\left[-\frac{\sigma}{2}(z_1 g_1 + z_2 g_2)^2\right]. \tag{18}$$

Hence, the mean energy density $\bar{U}(y, \eta)$ has the operator representation

$$U(y, \eta) = \left\{F\left(y, \eta, \frac{1}{i}\frac{\partial}{\partial z_1}, \frac{1}{i}\frac{\partial}{\partial z_2}\right) \exp\left[-\frac{\sigma}{2}(z_1 g_1 + z_2 g_2)^2\right]\right\}_{z_1=z_2=0}, \tag{19}$$

from which we can change to the more convenient integral representation by representing the generating functional by a Fourier transformation of its Fourier transformant:

$$K(z_1, z_2, g_1, g_2) = \frac{1}{2\pi\sigma\sqrt{g_1^2 g_2^2 - (g_1, g_2)^2}} \int dp_1 dp_2 \exp\left[-\frac{1}{2\sigma}\frac{(p_1 g_2 - p_2 g_1)^2}{g_1^2 g_2^2 - (g_1, g_2)^2} + ip_1 z_1 + ip_2 z_2\right]. \tag{20}$$

It is seen that when the operator F is applied to Eq. (20) under the integral, $\frac{1}{i}\frac{\partial}{\partial z_k}$ is replaced by p_k as a result of which Eq. (17) for the mean energy density takes the form

$$\bar{U}(y, \eta) = \frac{1}{2\pi\sigma\sqrt{g_1^2 g_2^2 - (g_1, g_2)^2}} \int dp_1 dp_2 \exp\left[-\frac{1}{2\sigma}\frac{(p_1 g_2 - p_2 g_1)^2}{g_1^2 g_2^2 - (g_1, g_2)^2}\right] F(y, \eta, p_1, p_2). \tag{21}$$

In the last expression we now have a double integral so that Eq. (21) for the mean energy density is in practical respects much more convenient than Eq. (12) in the form of an N-tuple integral.

Note that in view of the Cauchy inequality $(g_1, g_2)^2 \leq g_1^2 g_2^2$ and Eq. (21) does not lose any meaning for any g_1 and g_2.

§ 4. When the entrance beam has a plane wave front, the normal of which coincides with the optic axis, the function F can be written in the form

$$F = \Psi[a_{11}y + a_{12}\eta + (g_1, \lambda)]\,\delta[a_{21}y + a_{22}\eta + (g_2, \lambda)] \tag{22}$$

and the expression for the mean energy density $\bar{U}$ can be represented in a simpler form. Since we are only interested in the behavior of $\bar{U}$ in configuration space, using the presence of the δ-function we integrate (21) over the angles η:

$$\bar{U}(y) = \frac{1}{\sqrt{2\pi\sigma g^2 a_{22}^2}} \int dp \exp\left(-\frac{p^2}{2\sigma^2 g^2}\right) \Psi\left(\frac{y}{a_{22}} + p\right), \tag{23}$$

where $g_1 - (a_{12}/a_{22})\, g_2 = g$.

The last expression shows that the function Ψ can have a fairly general form, since it is integrated with a strong cutoff factor. In the general case we can obtain for Eq. (23) an asymptotic representation using, for example, the Laplace method [6]. For $\Psi(\tau) = \exp(-\tau^2/2)$, Eq. (23) is easily evaluated:

$$\bar{U}(y) = \frac{1}{\sqrt{a_{22}^2(1 + \sigma g^2)}} \exp\left[-\frac{y^2}{2}\frac{1}{a_{22}^2(1 + \sigma g^2)}\right]. \tag{24}$$

The coefficients a_{il}, g_1, g_2, and g which occur in Eqs. (21) and (23) depend on the number of lenses in the system N and on the parameters of the system α, l, and D. We will now calculate these coefficients.

§ 5. First we will calculate $(VP)^k$. In the notation given by Eqs. (3) we can write

$$VP = c_0\sigma_0 + (\vec{c}, \vec{\sigma}),$$

where

$$\vec{c} = (c_1, c_2, c_3), \quad \vec{\sigma} = (\sigma_1, \sigma_2, \sigma_3), \quad (\vec{c}, \vec{\sigma}) = c_1\sigma_1 + c_2\sigma_2 + c_3\sigma_3 \tag{25}$$

and

$$c_0 = \cos\varphi - \frac{L}{2}\sin\varphi, \quad c_1 = -\frac{L}{2}\cos\varphi, \quad c_2 = -i\left(\sin\varphi + \frac{L}{2}\cos\varphi\right), \quad c_3 = \frac{L}{2}\sin\varphi.$$

A consequence of relation (4) is

$$(\vec{c}, \vec{\sigma}) = \begin{cases} c^k, & k = 2m, \\ c^k(\vec{c}, \vec{\sigma}), & k = 2m+1, \end{cases}$$

$$m = 0, 1, 2, \ldots; \quad c = \sqrt{(\vec{c}, \vec{c})} = \sqrt{c_0^2 - 1},$$

which enables us to represent $(VP)^k$ in the form

$$(VP)^k = c_0^k\left[\sum_{\substack{\text{even}\\ m=0}}^{k}\binom{k}{m}(c/c_0)^m + \frac{(\vec{c}, \vec{\sigma})}{c}\sum_{\substack{\text{odd}\\ m=0}}^{k}\binom{k}{m}(c/c_0)^m\right], \tag{26}$$

where $\binom{k}{m}$ are binomial coefficients.

The sums of the form $\sum\limits_{\text{even}}\binom{k}{m}\beta^m$, $\sum\limits_{\text{odd}}\binom{k}{m}\beta^m$, which occur in Eq. (26), can be evaluated by introducing a generating function of the type $H(t) = h_1(t)\cdot h_2(\beta t)$, where $h_1(t)$, $h_2(t) = \operatorname{ch} t$, $\operatorname{sh} t$, and taking its derivative with respect to t of the k-th order. We thereby obtain

$$\begin{aligned} \sum_{\text{even}}\binom{k}{m}\beta^m &= \frac{1}{2}[(1+\beta)^k + (1-\beta)^k], \\ \sum_{\text{odd}}\binom{k}{m}\beta^m &= \frac{1}{2}[(1+\beta)^k - (1-\beta)^k]. \end{aligned} \tag{27}$$

Thermal gas lenses are usually short, so that $\sqrt{\alpha}\, l = \varphi \ll 1$, $\beta = c/c_0 = i\gamma$ in the neighborhood $\varphi = 0$, and γ is a real number. Then, introducing the complex number $\zeta = 1 + i\gamma$, we can write

$$(VP)^k = \frac{1}{2|\zeta|^k}\left[(\zeta^k - \bar{\zeta}^k) + (\vec{c}, \vec{\sigma})\frac{|\zeta|}{i\gamma}(\zeta^k - \bar{\zeta}^k)\right]. \tag{28}$$

The complexity of Eq. (28) is only apparent as can easily be shown by representing ζ^k in the form $\exp[k \ln|\zeta| + ik \arg\zeta]$. As a result we obtain

$$(VP)^k = \cos k\delta + (\vec{c}, \vec{\sigma})\frac{\zeta}{\gamma}\sin k\delta, \tag{29}$$

where $\delta = \arg\zeta$.

Putting $\cos k\delta = A^k$ and $(|\zeta|/\gamma)\sin k\delta = B^k$, we have for the matrices $\|a_{il}\|$ and $\|b_{il}^k\|$ the following expressions:

$$\begin{aligned} (VP)^k V = {} & \left[A^k\cos\varphi - B^k\sin\varphi\left(\sin\varphi + \frac{L}{2}\cos\varphi\right)\right]\sigma_0 - \\ & - \left[B^k\frac{L}{2}\right]\sigma_1 - i\left[A^k\sin\varphi + B^k\cos\varphi\left(\sin\varphi + \frac{L}{2}\cos\varphi\right)\right]\sigma_2, \\ (VP)^k(1-V) = {} & \left[A^k(1-\cos\varphi) + B^k\sin\varphi\left(\sin\varphi + \frac{L}{2}\cos\varphi\right)\right]\sigma_0 + \\ & + \left[B^k\frac{L}{2}(1-\cos\varphi)\right]\sigma_1 + i\left[A^k\sin\varphi - B^k(1-\cos\varphi)\left(\sin\varphi + \frac{L}{2}\cos\varphi\right)\right]\sigma_2 + \left[B^k\frac{L}{2}\sin\varphi\right]\sigma_3. \end{aligned}$$

It is not now difficult to obtain the matrix elements we require; in particular, the components of the vectors g_i are given by

$$g_1^k = b_{11}^k = A^k(1-\cos\varphi) + B^k\sin\varphi\left(\sin\varphi + \frac{L}{2}\cos\varphi + \frac{L}{2}\right),$$
$$g_2^k = b_{21}^k = -A^k\sin\varphi + B^k(1-\cos\varphi)\left(\sin\varphi + \frac{L}{2}\cos\varphi + \frac{L}{2}\right). \tag{30}$$

Expressions (21) and (23) for the mean energy density of the beam only contain squares of the vectors g_i and their scalar product. We will obtain these quantities.

Representing g_i in the form

$$g_i^k = d_i A^k + q_i B^k;$$

we obtain

$$(g_i, g_j) = d_i d_j \sum_{k=0}^{N-1}(A^k)^2 + (d_i q_j + d_j q_i)\sum_{k=0}^{N-1} A^k B^k + q_i q_j \sum_{k=0}^{N-1}(B^k)^2, \quad i, j = 1, 2. \tag{31}$$

The sums in Eq. (31) can be found by representing $\cos k\delta$ and $\sin k\delta$ by means of the Euler equations. Neglecting terms that are bounded as $N \to \infty$ we can write

$$(g_i, g_j) \approx \frac{N}{2}(d_i d_j + q_i q_j) \tag{32}$$

and we then obtain the following expression for the mean energy density:

$$\bar{U}(y, \eta) \approx \frac{1}{\pi\sigma N\sqrt{d_1^2 q_2^2 + d_2^2 q_1^2 - 2d_1 d_2 q_1 q_2}} \int dp_1 dp_2 F(y, \eta, p_1, p_2) \times$$
$$\times \exp\left[-\frac{1}{\sigma N}\frac{p_1^2(d_2^2 + q_2^2) + p_2^2(d_1^2 + q_1^2) - 2p_1 p_2(d_1 d_2 + q_1 q_2)}{d_1^2 q_2^2 + d_2^2 q_1^2 - 2d_1 d_2 q_1 q_2}\right], \tag{33}$$

where

$$d_1 = 1 - \cos\varphi; \quad q_1 = \frac{|\zeta|}{\gamma}\sin\varphi\left(\sin\varphi + \frac{L}{2}\cos\varphi + \frac{L}{2}\right);$$
$$d_2 = -\sin\varphi; \quad q_2 = \frac{|\zeta|}{\gamma}(1-\cos\varphi)\left(\sin\varphi + \frac{L}{2}\cos\varphi + \frac{L}{2}\right)$$

and

$$\varphi = \sqrt{\alpha}\, l, \quad L = \sqrt{\alpha}\, D.$$

The mean energy density in the case of a plane entering beam [expression (23)] now has the form

$$\bar{U}(y) \approx \frac{1}{\sqrt{\pi\sigma N a_{22}^2\left[d_1^2 + q_1^2 + \frac{a_{12}^2}{a_{22}^2}(d_2^2 + q_2^2) - 2\frac{a_{12}^2}{a_{22}^2}(d_1 d_2 + q_1 q_2)\right]}} \times$$
$$\times \int dp\Psi\left(\frac{y}{a_{22}} + p\right)\exp\left[-\frac{1}{\sigma N}\cdot\frac{1}{d_1^2 + q_1^2 + \frac{a_{12}^2}{a_{22}^2}(d_2^2 + q_2^2) - 2\frac{a_{12}^2}{a_{22}^2}(d_1 d_2 + q_1 q_2)}\right]. \tag{34}$$

Expressions (21) and (23) or (33) and (24) enable us to calculate in a simple manner the mean energy W at the exit of the last lens,

$$W = \int_{-a}^{a} dy \bar{U}(y),$$

and to find the loss in energy in the system, i.e., $1 - W$.

NOTATION

U, light energy density; Φ, initial light energy distribution; ρ, distribution function of the random displacements; l, length of a lens; D, distance between lenses; α, specific convergence of the structure in a lens; σ_0, unit 2 × 2 matrix; σ_i, Pauli matrix; λ, random displacements of the lenses; σ, the variance of the random displacements; N, number of lenses in the light guide.

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CALCULATING ANGULAR RADIATION COEFFICIENTS BY THE METHOD OF FLOW ALGEBRA

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A method is described for the calculation of mean angular radiation coefficients in two-dimensional systems consisting of any number of plane surfaces, including systems in which two adjacent surfaces form a concave part of the contour. It is shown that for the calculation it is sufficient to know the coordinates of all zone boundaries and the characteristic point of the system.

In calculations of radiative heat exchange between surfaces of a system infinitely stretched out in one direction (a two-dimensional system), the method of flow algebra is widely used for the determination of mean angular radiation coefficients [1, 2]. This method is often called the method of stretched strings, the enveloping curves method, and the algebraic method. In conformity with the notation in Fig. 1, the angular coefficient between two terminal surfaces F_1 and F_2 is given by the simple algebraic expression

$$\varphi_{1,2} = \frac{(AC + BTD) - (AD + BTKC)}{2AB}, \tag{1}$$

where AC, BTD etc., are the lengths of the elastic strings stretched between the corresponding boundaries of the surfaces F_1 and F_2.

It should be noted that the determination of the lengths of elastic strings in systems with a large number of zones, particularly in the case of calculations with many variants, gives rise to fundamental difficulties and, as a rule, necessitates the use of a computer. Here it is desirable to describe the system by a minimum number of initial values and to calculate the elements of the matrix of angular radiation coefficients according to a universal relation.

The objective of the present work is application of the method of flow algebra for the calculation of angular coefficients in two-dimensional systems of plane surfaces.

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